**THREE DIMENSIONAL GEOMETRY**

* This deals with closed figure whose interior cannot be seen e.g. Pyramids, cuboids, prisms, Tetrahedrons etc.
* Such figures consist of flat surfaces which are triangular, rectangular, pentagonal etc.
* The line where the flat surfaces meet is called an **edge**
* A flat surface is called a **plane**
* The lengths of lines can be got using Pythagoras҆ theorem or trigonometry of a triangle
* The angle between a line and a plane is the same as the angle between the line and its shadow on the plane.
* The angle between two planes lies at the mid-point of the common line of the planes or the common point.
* In finding the angle between two planes, the planes are bisected from the mid-point of the common line or the common point.

**PYRAMIDS**

* It is a closed figure with flat surfaces resting on a flat base which is a square, rectangular, pentagon
* When opened, a pyramid gives the following possible nets

Or

**Q. 1.**

The figure below shows a rectangular pyramid with each slant edge 26 cm long

16 cm

12 cm

26 cm

D

B

C

O

V

A

Calculate the;

1. Height OV
2. Length AV
3. Angle between line AV and the base ABCD
4. Angle between the planes ABV and ABCD
5. Angle between planes ABV and DCV
6. Angle between the planes ADV and BCV
7. Volume of the pyramid
8. Surface area of the prism

**Q. 2**

The figure below shows a square based pyramid ABCDV in

which AB = BC = 6 cm and OV = 10 cm

6 cm

6 cm

10 cm

A

C

D

O

V

B

Find the;

i)length VA

ii) angle between the AV and plane ABCD

iii) angle between planes VBC and ABCD

iv) angle between planes VBC and VAD

v) volume of the pyramid

Q.3

The figure below shows a square based pyramid with

equilateral triangular slant faces . Given that AB = 5 cm.

5 cm

A

C

D

O

V

B

Calculate the;

i)height of the triangular faces

ii) length AC

iii) angle between VA and ABCD

iV)angle between VAD and ABCD

V) angle between VAB and VBC

Q.4

PQRSV is a pyramid with a vertical plane VQR. PQRS is a

square with length 40 cm and VQR is an equilateral triangle

P

R

Q

S

V

O

Find the;

i)lengths OV and PV

(**Hint; join P to O and use triangle POV**)

ii) angle between line PV and plane PQRS

iii) angle between the planes VSP and PQRS

iv) total surface area

v) volume of the pyramid

Q.5

The figure shows a right pyramid with PQ = 24 cm , QR = 18

Cm and angle TRO = 65o

24 cm

P

R

S

O

T

Q

18 cm

65o

Find the;

i) Length TO and TR

ii) Angle between the planes TQR and PQRS

iii) Angle between the planes TQR and TSP

iv) Volume of the prism

Q.6

The figure below shows a pyramid whose base ABCD is a

rhombus of side 5 cm and whose acute angle is 60o.

AE = DE = CE = BE = 8 cm. F is a point of intersection of the

5 cm

A

C

D

F

E

B

60o

5 cm

8 cm

diagonals of the rhombus.

Find the;

i) length EF

ii) angle AEB

iii)angle each of the slanting faces makes with base .

Q.7

The figure below shows a rectangular based pyramid of sides 8 cm by 6 cm in which AE = DE = BE = CE = 6 cm. F is the point of intersection of the diagonals and E is a point such that 

8 cm

A

C

D

F

E

B

6 cm

6 cm

G

Find the;

i)angle AEC

ii) length EF and AG

iii) angle which each of the slanting planes makes

with the base

Q.8

The figure below shows a net of pyramid consisting of a square of side 12 cm and four congruent isosceles triangles

D

A

B

C

Given that AB = CD = 40 cm, calculate the;

1. i)height of the vertex of the pyramid from the square

base

ii) angle between the triangular base and the base of

the pyramid

iii) volume of the pyramid

1. If the pyramid is cut horizontally at a vertical height of 2.6 cm from the square base, and the upper part of the pyramid containing the vertex is thrown away, find the volume of the remaining solid.

Q.9

The figure below shows a net of a pyramid consisting a square of side 6 cm and four congruent isosceles triangles. The distance AB = 20 cm.

A

B

Calculate the;

a) Total surface area of the pyramid

b) Perpendicular height of the pyramid formed when the

net is folded

c) angle of inclination of the triangular face to the base of

the pyramid.

Q.10

The figure below is a net of a pyramid consisting of a square of side 12 cm and four congruent isosceles triangles each of vertical height 10 cm.

Calculate the;

1. Vertical height of the pyramid
2. Angle each sloping face makes with the base
3. Angle between the opposite sloping faces
4. Volume of the pyramid

Q.11

The figure shows the wedge with a rectangular base ABCD and a vertical rectangular face DCFE. EF = 20 cm, BC = cm and FC = 5 cm

A

F

E

D

C

B

10 cm

5 cm

20 cm

Calculate the;

i)angle between planes ABCD and ABFE

ii) length of AF and its angle of inclination to the horizontal

iii)volume of the wedge.

Q.12

The figure below shows the a prism ABCDEF with an isosceles right angled triangle as cross section and horizontal rectangular base ABCD

A

F

E

D

B

C

10 cm

15 cm

Calculate the;

i) lengths AF and BE

ii) angle between BE and the base

iii) volume of the prism

Q.13

A right prism ABCDEF is 12 cm long and its cross section is an equilateral triangle of side 6 cm. The base ABCD is rectangular and horizontal.

A

F

E

D

B

C

6 cm

12 cm

Calculate the;

i) volume of the triangular face

ii) length EM where M is the mid-point of AB

iii)angle between EM and the base

Q.14

The diagram below shows a solid object with a regular pentagonal base of side 20 cm and centre O. The vertex V is vertically above O and VO = 30 cm.

A

V

D

D

C

B

O

20 cm

Find the;

i)angle BCO

ii) length OC

iii)length of VC and the angle its inclination to the base

Q.15

The figure below shows a right pyramid with a rectangular base measuring 80 cm by 60 cm whose top part FGHEV is cut off. Given that FGHE is parallel to the base PQRS and slant length of the remaining part RG= 30 cm.

P

Q

R

S

B

E

F

G

H

A

V

80 cm

60 cm

30 cm

Given that 4VA = 3 BV, calculate the;

i) lengths EF, FG and VG.

ii) height BV of the pyramid

iii) volume of the frustrum PQRSHEFG.

**TETRAHEDRON**

It is a closed figure with four triangular faces. A regular tetrahedron has all its sides equal.

**Nets of tetrahedron**

Q.1

The figure below shows a net of a regular tetrahedron of side 8 cm.

8 cm

8 cm

a) Draw the tetrahedron

b) Calculate the;

i) height of each triangular face.

ii) total surface area of the tetrahedron

Q.2

The figure below shows a tetrahedron VABCD consisting of right angled triangular faces. AV = AC = 8 cm and AB = 6 cm.

A

V

B

8 cm

6 cm

C

Given that angle VAB = 90o,

a) Calculate the;

i) Lengths VB, BC and VC.

ii) Volume of the tetrahedron

b) Draw a net of the tetrahedron and hence find its total

surface area.

Q.3

The figure below shows a regular tetrahedron PQRS of side 12 cm.

P

Q

R

S

O

12 cm

M

Calculate the;

i) lengths OM, OR and OS

ii) angle each slant edge makes with the base

iii) angle between the planes PQR and PQS

iv) volume of the tetrahedron

1. total surface area of the figure.

Q.4

A regular tetrahedron PQRV is of side 8 cm and the vertex V is directly above the center of the base.

a) Draw the tetrahedron

b)Calculate the;

i) height of V above the base

ii) angle each slant edge makes with the base

iii) angle each slant face makes with the base

iv) volume

CUBOIDS AND CUBES

A cuboid is a closed figure bounded by six rectangles.

A cube is a cuboid having all its six faces as squares.

**Possible nets**

For closed cuboid

**For Open cuboid**

**Q.1**

The figure below shows a rectangular box measuring 30 cm by 40 cm by 20 cm. X is the mid-point of BC

A

B

C

D

P

Q

R

S

X

Calculate the;

a) lengths AQ, BR , AC and AR

b) angle between AR and the base ABCD

c) angle between the planes;

i) ABRS and ABCD

ii) ADRQ and ABCD

iii) ACQ and ABCD

iv) PSX and PQRS

v) QCS and BCRQ

vi) DPR and ADSP

d) angle QAR

Q.2

The figure below shows a cuboid ABCDYPQX onto which a wedge PQXYSR of vertical height 10 cm sits on. Given that AB = 40 cm and BC = 30 cm, CX = 8 cm.

A

B

C

D

P

Q

X

Y

S

40 cm

30 cm

8 cm

R

Find the;

a)lengths QR and PR

b) angle QRC

c) angle between the planes ABCD and PQRS

d) angle of inclination of PR to the horizontal

e) angle of inclination of AR to the horizontal

Q.3

The figure below shows a cube ABCDEFGH of side 8 cm and

EM = MF. A tetrahedron AMHE is cut off from the cube.

8 cm

D

G

H

E

A

F

B

C

M

Calculate the;

i)area of triangle HAM

ii) angle between planes HAM and AEHD

iii) volume of the remaining part of the cube after the

tetrahedron has been cut off